|  |
| --- |
| Abstract |
| **Arbelos is the region bounded by the three semicircles in the figure below Pappus chain is the chain of circles inscribed in the arbelos. Pappus of Alexandria proved a theorem in the 4th century A.D., which states that the height from the center of the *nth* inscribed circle is equal to *n* times the diameter of that circle (ScienceBuddies.org, 2017).**    **Pappus used Euclidean geometry to prove his theorem. Jakob Steiner invented *circle inversion* in the 19th century, which made life easier to prove complex theorems. My objective is to develop an open-source software application to simulate circle inversion and prove Pappus' theorem.** |

|  |
| --- |
| Problem |
| **Prove that the height from the center of the *nth* inscribed circle in the Pappus chain is equal to *n* times the diameter of that circle by using *circle inversion*.** |

|  |
| --- |
| Introduction |
| **Inversion transforms a point *P* to a point *P′* with respect to an inversion circle. An inversion circle can be described by its inversion center *O* and inversion radius *r*. For *P′* to be the inverse of point *P*, the points *O*, *P*, and *P′* must be on the same line, and *OQ* and *QP′* must be perpendicular to each other. From similar triangles: *OP.OP′ = r2* (WolframMathWorld, 2017)**    **Inversion of a circle transforms all points on the original circle with respect to an inversion circle. The outcome will be a circle or line (Tom Davis, 2011).**  **Possible scenarios for circle inversion:**   * **If the original circle doesn’t pass through the center of the inversion circle, it will be inverted to a circle.** * **If the original circle passes through the center of the inversion circle, it will be inverted to a line.**   **Homothety, also known as dilation or central similarity, is a transformation of a shape, which sends each point *M* on the original shape to a point *M′* on the line *OM* such that *OM′ = k.OM*, and *k* is a non-zero number (EncyclopediaOfMath, 2014).** |

|  |
| --- |
| Materials & Methods |
| **Hardware: MacBook Pro**  **Software: HTML5, JavaScript, Brackets (text editor)**  **Source Code: 1100 lines of client-side code**  **License: Open-source for free public use**   * **Developed a web page to invert a point, a circle, and a Pappus chain** * **Converted screen coordinates to standard math coordinates** * **Created tabstrip with CSS classes in HTML5 without using JavaScript** * **Used the HTML5 <canvas> tag as the graphics container** * **Used JavaScript to draw shapes and handle user requests** * **Provided templates to load predefined data and invert automatically** * **Allowed the user to enter custom values and to run their own inversions** |

|  |
| --- |
| Discussion |
| **Each original circle in the Pappus chain and its inverted circle between the lines are homothetic with the center of homothety being the center of the blue inversion circle. Connecting the center of homothety to the centers of the original and inverted circles will create similar triangles as shown below.**    **Let *dn* be the diameter and *hn* be the height from the center of the original circle *n* in the Pappus chain. Similarly, let *d'n* be the diameter and *h'n*be the height from the center of the inverted circle *n* between the lines. Since the circles between the lines are identical, we can easily observe that:**    **From similarity:**    **Therefore:**   |  |  |  | | --- | --- | --- | |  | **or** |  |   **This simply proves Pappus’ theorem, i.e. the height from the center of the *nth* inscribed circle is equal to *n* times the diameter of that circle.** |

|  |
| --- |
| Conclusion |
| **Circle inversion can be used to solve difficult problems such as proving Pappus’ theorem by visually showing similarity between circles and their inversions.**  **I developed a computer program with graphics features to simulate circle inversion and Pappus chain visually, while also showing the calculations. I created a web page in HTML5 and JavaScript, which runs in the latest browsers on all computers and mobile devices.**  **I share this application as an open-source tool with anyone interested in math, and specifically in circle inversion and Pappus chain. Please feel free to use the application and get the source code from my web page at** [**http://sukarablog.weebly.com**](http://sukarablog.weebly.com)**.** |

|  |
| --- |
| Further Research |
| **This project builds the basis for inverting a point or a circle with respect to an inversion circle. However, it can be extended to the inversion of other shapes to analyze the effects of circle inversion. It can also be improved to invert shapes and objects with respect to a sphere in 3-D since most of the equations will be very similar to 2-D.** |

|  |
| --- |
| Results |
| **Runs in the latest browsers on all computers and mobile devices** |

|  |
| --- |
| Circle |

|  |
| --- |
| Inversion |

|  |
| --- |
| & |

|  |
| --- |
| Pappus |

|  |
| --- |
| Chain |